

Math Analysis: Spring Final (Review)

- 1) A total of \$12,000 is invested in two corporate bonds that pay 10.5% and 12% simple interest. The annual interest is \$1380. How much is invested in each bond?

\$4000 at 10.5% \$8000 at 12%

Numbers 2-3: Write the partial fraction decomposition for the rational expression. Check your result algebraically and graphically using a graphing utility.

2) $\frac{4x^2-1}{2x(x+1)^2} = \frac{1}{2} \left[-\frac{1}{x} + \frac{5}{x+1} - \frac{3}{(x+1)^2} \right]$ 3) $\frac{x^2-1}{x(x^2+1)} = \frac{x^2-1}{x(x^2+1)} = -\frac{1}{x} + \frac{2x}{x^2+1}$

- 4) A merchant plans to sell two models of home computers at \$250 and \$400, respectively. The \$250 model yields a profit of \$45 and the \$400 model yields a profit of \$50. The merchant estimates the total monthly demand will not exceed 250 units. Find the number of units of each model that should be stocked in order to maximize profit. Assume that the merchant does not want to invest more than \$70,000 in computer inventory.

200 units at \$250 50 units at \$400

- 5) Find the n th term of the geometric sequence. $a=1000$, $r=1.005$, $n=60$

$a_n = 1000(1.005)^{59}$

- 6) Find the sum of the infinite geometric sequence. $\sum_{n=0}^{\infty} 2\left(\frac{2}{3}\right)^n = 2 + \frac{4}{3} + \frac{8}{9} + \frac{16}{27} + \dots$

Sum = 6

7) Find P_{k+1} $P_k = \frac{k^2(k+1)^2}{4} = \frac{(k+1)^2(k+2)^2}{4}$

- 8) $(5x-3y)^{10}$ Determine the coefficient of the 8th term. $-32805000x^3y^7$

- 9) Four people are lining up for a ride on a toboggan, but only two of the four are willing to take the first position. With that constraint, in how many ways can the four people be seated in the toboggan? 12

- 10) In a certain state the automobile license plates consist of two letters followed by a four-digit number. How many distinct license plates can be formed? 6,760,000

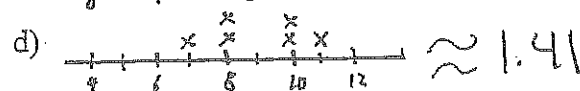
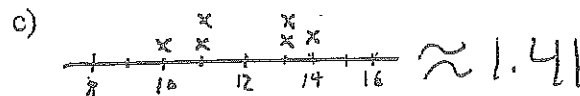
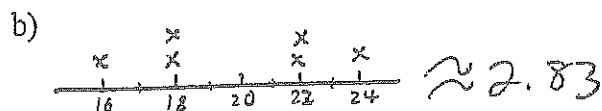
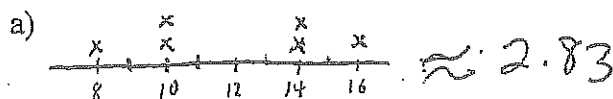
- 11) In how many orders can three girls and two boys walk through a doorway single-file if

- a) there are no restrictions? 120
b) the boys go before the girls? 12
c) the girls go before the boys? 12

- 12) A committee composed of three graduate students and two undergraduate students is to be selected from a group of eight graduates and five undergraduates. How many different committees can be formed? 560

- 13) Find the probability that a card drawn from a standard 52-card deck will be a 6 or less. $\frac{9}{13}$

- 14) Determine the mean and standard deviation of each set.



- 15) Find the sum.

$$\sum_{k=2}^5 4k = 56$$

- 16) Find the sum of the first 100 positive multiples of 5.

$$25,250$$

- 17) Find the vertex. $y^2 + 4y + 8x - 12 = 0$

$$(2, -2)$$

Numbers 18-19: Find an equation of the specified hyperbola.

- 18) Vertices: (2,0), (6,0) Foci: (0,0), (8,0)

$$\frac{(x-4)^2}{4} - \frac{y^2}{12} = 1$$

- 19) Vertices: (4,1), (4,9) Foci: (4,0), (4,10)

$$\frac{(y-5)^2}{16} - \frac{(x-4)^2}{9} = 1$$

- 20) Sketch the curve of the parametric equations (indicate the direction of the curve). Use a graphing utility to confirm your result. Eliminate the parameter and write a rectangular equation whose graph represents the curve.

$$x^2 + y^2 = 9$$

$$X = 3\cos T \quad Y = 3\sin T$$



- 21) Plot the point in polar coordinates and find the corresponding rectangular coordinates for the point.



Numbers 22-23: Plot the point and find two sets of polar coordinates for the point for $0 \leq T < 2\pi$

- 22) $(-3, -3)$
 $(3\sqrt{2}, \frac{5\pi}{4})$ $(-3\sqrt{2}, \frac{\pi}{4})$

- 23) $(-3^{1/2}, -3^{1/2})$
 $(\sqrt{6}, \frac{5\pi}{4})$ $(-\sqrt{6}, \frac{\pi}{4})$

Numbers 24-25: Convert the polar equation to rectangular form.

- 24) $r = 4\sin T$

$$x^2 + y^2 - 4y = 0$$

- 25) $r = 4\cos T$

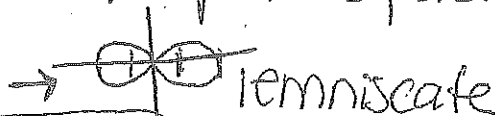
$$x^2 + y^2 - 4x = 0$$

- 26) Test for symmetry with respect to $T = (\pi)/2$, the polar axis, and the pole.

$$r^2 = 25 \sin 2T$$

Symmetric w/ respect to pole.

- 27) Sketch the graph of the polar equation $r^2 = 4 \cos 2T$



- 28) Find an equation of the ellipse.

$$\frac{(x-2)^2}{4} + \frac{(y-1)^2}{1} = 1$$

Vertices: (0,1), (4,1) Minor Axis Endpoints: (2,0), (2,2)

- 29) Find an equation of the hyperbola.

Vertices: (2,2), (-2,2) Foci: (4,2), (-4,2)

$$\frac{x^2}{4} - \frac{(y-2)^2}{12} = 1$$

Numbers 30-31: Sketch v and find its component form. (Assume T is measured counterclockwise from the x -axis to the vector.)

30) $\|v\| = 5/2$

$T = 45$ degrees

$$\left(\frac{5\sqrt{2}}{4}, \frac{5\sqrt{2}}{4} \right)$$

31) $\|v\| = 3 \cdot 2^{1/2}$

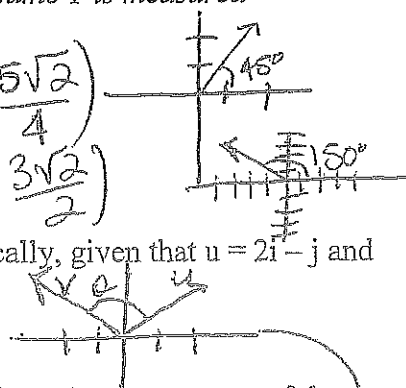
$T = 150$ degrees

$$\left(-\frac{3\sqrt{6}}{2}, \frac{3\sqrt{2}}{2} \right)$$

- 32) Find the component form of v and sketch geometrically, given that $u = 2i - j$ and $w = i + 2j$

$$v = u - 2w$$

$$-5j$$



- 33) Use a graphing utility to sketch the vectors and find the degree measure of the angle between the vectors.

$$u = 3i + 4j$$

$$v = -7i + 5j$$

$$\approx 91.33^\circ$$

- 34) Find the standard equation of the sphere: Center(1, 2, 3) Radius=5

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = 25$$

- 35) Find a unit vector in the direction of u :

$$u = 8i + 3j - k$$

$$\frac{1}{\sqrt{74}}(8i + 3j - k)$$

- 36) Determine whether u and v are orthogonal, parallel, or neither.

$$u = \langle -12, 6, 15 \rangle$$

$$v = \langle 8, -4, -10 \rangle$$

parallel

- 37) Find $u \times v$ and show that is orthogonal to both u and v .

$$u = 6i + 2j + k$$

$$v = i + 3j - 2k$$

$$= -7i + 13j + 16k$$

- 38) Find the limit by direct substitution.



$$\lim_{x \rightarrow 5} (x^2 - 5x - 25) = 25$$

Numbers 39-40: Find the limit (if it exists).

39) $\lim_{x \rightarrow -\infty} \frac{4x-3}{2x+1} = 2$

40) $\lim_{x \rightarrow \infty} \frac{2x^2-6}{(x-1)^2} = 2$